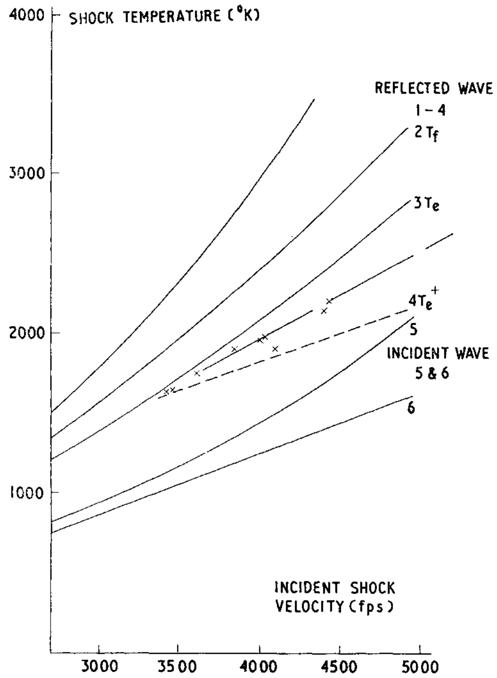

Fig 2 Comparison of experimentally determined temperatures with those calculated making a variety of basic assumptions

examined, the variation in temperature in the first 3 in from the reflecting face was within the error of measurement With an end-cap instead of a plug, the temperature was too low to be measured, indicating the arrival of the contact surface At a pressure ratio of 180, the calculated "slug" length, assuming constant specific heats and no reaction, was 13 8 in ; the estimated length was 5 5 in This result is in agreement with the conclusions of Faizullov et al ,[5] but the ratio of calculated to measured "slug" length was greater in the work reported here because of the effect of chemical reaction

Discussion

In order that a measurement relating to C_2 may be made, some reaction is first necessary to produce this entity, so that the temperature measured relates to a reaction product This method, however, possesses the advantage that the system is kept free from additives It was found that, with the concentrations used, the C_2 temperature was unaffected by the addition of sodium chloride

The degree of agreement between the C_2 and sodium-line reversal temperatures suggests that there is a pronounced degree of equilibrium in the driven gas This agreement was independent of pressure of the system in the range examined

It was found that in this reacting system, as in chemically inert systems, incident shock velocity was a major way of characterizing an experiment Measured temperatures have, therefore, been plotted against this value, as shown in Fig 2 Curves have also been plotted of temperatures calculated by making certain assumptions regarding the properties of the heated gas These are as follows:

Curve 1: Constant specific heats assumed throughout

Curve 2: Allowance made for variation of specific heat with temperature in the incident shock only (T_f)

Curve 3: Allowance made in both incident and reflected shocks for variation of specific heat with temperature (T)

Curve 4: As in curve 3, assuming that the degree of decomposition was according to $k = 10^{13} \exp(-85/RT)s^{-1}$ This temperature to be denoted by T^+

Of the calculated temperatures, obviously that from curve 1 is incorrect T_f will be satisfactory only if this temperature is maintained long enough for chemical reaction to occur and to do so before attainment of thermal equilibrium However, the rate of relaxation derived from acoustic absorption data is approximately 100 times faster in this temperature range than the rate of decomposition of methane Further divergence from T_f will occur because of the period between heating by the incident and reflected shock, during which some thermal equilibration will occur

In curve 3, the establishment of thermal equilibrium before appreciable reaction occurs is assumed It would be expected, therefore, that T will be close to measured values The temperature T^+ (curve 4) is an instantaneous value at a time corresponding to a chosen conversion It is the temperature to which the system falls after abstraction of heat for the endothermic decomposition

In Fig 2, the measured temperatures fall between T and T^+ There is no tendency for them to fall closer to T^+ at higher temperatures, which might be so if the measured temperature were entirely dependent on the endothermicity of the reaction It is of interest to note that there is close agreement between the sodium and C_2 reversal temperatures, rather than separation in values wherein the former would show agreement with T_e and the latter with T^+

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Maximum Total Pressure Recovery across a System of *n* Shock Waves

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Nomenclature

a = $4\gamma/(\gamma+1)^2$
b = $(\gamma^2+1)/\gamma$
f_i = $[(\gamma+1)/(\gamma-1)][(y_i-1)/y_i]$
g_i = $[(\gamma-1)/(\gamma+1)]\{[4\gamma/(\gamma+1)^2]\ y_i - 1\}^{-1}$
h = $\dfrac{(\gamma-1)M_{n-1}^4 + [(5-\gamma)/2]M_{n-1}^2 - 1}{[(3\gamma+1)/\gamma]M_{n-1}^2 - 1}$
M_i = Mach number
n = number of shock waves
P_i = pressure
P_{T_i} = total pressure
x_i = $1 + [(\gamma-1)/2]M_i^2$
y_i = $1 + [(\gamma-1)/2]M_i^2 \sin^2\omega_i$
γ = ratio of specific heats
ω_i = shock wave angle
ω = shock wave angle for sonic Mach number downstream

1 Introduction

In 1944 Oswatitsch[1] published a classic paper on the total pressure recovery across a system of n shock waves The shocks were arranged so that the Mach number downstream

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Table 1 Numerical results for Oswatitsch shock systems

$n = 2$			$n = 3$			$n = 4$		
M_0	$M_0 \sin\omega_0$	(P_{T_2}/P_{T_0})	M_0	$M_0 \sin\omega_0$	(P_{T_3}/P_{T_0})	M_0	$M_0 \sin\omega_0$	(P_{T_4}/P_{T_0})
1 0	1 000	1 000	1 0	1 000	1 000	1 0	1 000	1 000
1 5	1 215	0 9867	1 5	1 125	0 9950	1 5	1 089	0 9973
2 0	1 470	0 9026	2 0	1 279	0 9567	2 0	1 196	0 9766
2 5	1 724	0 7500	2 5	1 443	0 8663	2 5	1 311	0 9217
3 0	1 966	0 5808	3 0	1 600	0 7430	3 0	1 424	0 8367
3 5	2 198	0 4302	3 5	1 752	0 6091	3 5	1 538	0 7292
4 0	2 418	0 3184	4 0	1 892	0 4879	4 0	1 639	0 6241
4 5	2 620	0 2286	4 5	2 023	0 3846	4 5	1 739	0 5200
5 0	2 816	0 1662	5 0	2 152	0 2986	5 0	1 832	0 4302

of any particulai shock was equal to the Mach number upstream of the following shock Foi each wave in the system, it was shown how its total pressure recoveiy (P_{Ti+1}/P_{Ti}) should be selected so that the over-all product (P_{Tn}/P_{T0}) should be a maximum In order to avoid a trivial result, Oswatitsch found it necessary to choose the last wave in the system to be of finite strength, and, in fact, he assumed that it was a normal shock The analytical method that he used was the well-known Lagrange technique of undetermined multipliers [2] The practical application, which Oswatitsch made of this work, was to supersonic air intakes It is impoitant in this connection that the Mach number downstieam of the shock system should be subsonic, and, because Oswatitsch assumed that the last shock was normal, his theoiy clearly met this requirement

In the present paper, the Oswatitsch problem and its solution aie stated, and some numeiical results computed from the theoiy are quoted The problem is then studied from a different approach, using the theorem[3] on the equality of the arithmetic and geometric means as the condition for a maximum This simple theorem simplifies the solution very considerably and enables one to solve the more general problem where the final shock is not restricted to being normal The algebraic expressions obtained are also considerably simpler than the corresponding Oswatitsch expressions, and, moieover, when numerical results aie computed from them, a significantly higher total pressure recovery is obtained

2 The Oswatitsch Problem

The pioblem can be stated as follows: Given a system of n consecutive shock waves, comprising one normal and $(n - 1)$ oblique shocks, how should the individual pressure recoveries (P_{Ti+1}/P_{Ti}) of the shock be chosen so that theii product (P_{Tn}/P_{T_0}) is a maximum?

$$\frac{P_{T_n}}{P_{T_0}} = \prod_{i=0}^{n-1} \frac{P_{T_{i+1}}}{P_{T_i}} = \prod_{i=0}^{n-1} f_i^{\gamma/(\gamma-1)}\, g_i^{1/(\gamma-1)}$$

The consecutive arrangement requires the continuity conditions

$$x_{i+1} = x_i f_i g_i \qquad i = 0, 1 \quad (n - 2) \tag{1}$$

whereas the normal shock condition is expressed by

$$x_{n-1} = y_{n-1} \tag{2}$$

Equations (1) and (2) are the constraints on the problem By an elegant application of I agrange technique, Oswatitsch obtained the following solution:

$$y_0 = y_i \qquad i = 0, 1 \quad (n - 2) \tag{3}$$

$$\frac{x_0}{x_{n-1}} = \left[\frac{y_i(ay_0 - 1)}{y_0 - 1}\right]^{n-1} \tag{4}$$

$$y_0 = \tfrac{1}{2}[1 + h + (1 - bh + h^2)^{1/2}] \tag{5}$$

Now because (P_{Ti+1}/P_{Ti}), (P_{i+1}/P_i) are functions only of y_i, then Eq (3) implies that

$$P_{T_1}/P_{T_0} = P_{Ti+1}/P_{Ti} \qquad i = 0,1 \quad (n - 2)$$

and

$$P_1/P_0 = P_{i+1}/P_i \qquad i = 0, 1 \quad (n - 2)$$

Similar results can also be deduced for the density ratios, entropy rise, and so on Equations (3, 4, and 5) define a particular solution when (M_0, n) are given Some numerical results that have been computed from these equations are shown in Table 1 and in Fig 1 Further results are given in Ref 4

3 Details of a Simpler Approach

Consider again the problem discussed by Oswatitsch, but this time suppose that one drops the restriction that the final shock should be noimal to the flow Appeal is now made to the following theorem

Fundamental theorem

To partition a given positive quantity S into n positive parts $S = s_1 + s_2 \quad + s_n$, so that the product $P = s_1 s_2 \quad s_n$

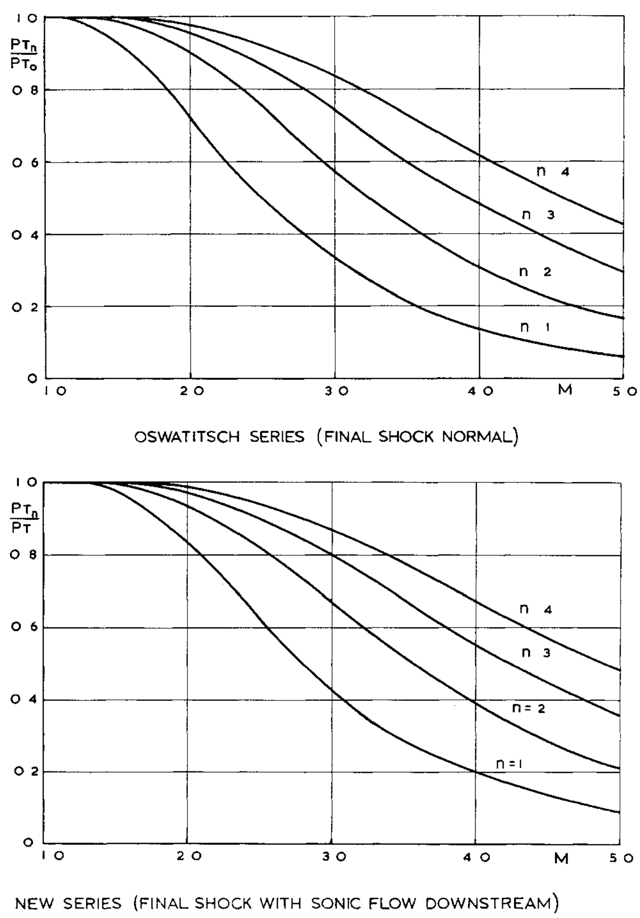

Fig 1 Total pressure recovery of two series of shock systems

Table 2 Numerical results for new shock systems

$n = 2$			$n = 3$			$n = 4$		
M_0	$M_0 \sin\omega_0$	(P_{T_2}/P_{T_0})	M_0	$M_0 \sin\omega_0$	(P_{T_3}/P_{T_0})	M_0	$M_0 \sin\omega_0$	(P_{T_4}/P_{T_0})
1 0	1 000	1 000	1 0	1 000	1 000	1 0	1 000	1 000
1 5	1 154	0 9928	1 5	1 100	0 9967	1 5	1 074	0 9980
2 0	1 353	0 9390	2 0	1 227	0 9700	2 0	1 166	0 9825
2 5	1 573	0 8188	2 5	1 367	0 9015	2 5	1 267	0 9401
3 0	1 795	0 6641	3 0	1 510	0 7956	3 0	1 371	0 8675
3 5	2 010	0 5129	3 5	1 649	0 6732	3 5	1 473	0 7744
4 0	2 216	0 3854	4 0	1 782	0 5526	4 0	1 571	0 6732
4 5	2 412	0 2861	4 5	1 908	0 4453	4 5	1 663	0 5750
5 0	2 598	0 2123	5 0	2 027	0 3552	5 0	1 751	0 4840

will be a maximum Then the required condition is that $g = m$, where m is the arithmetic mean, namely,

$$m = (s_1 + s_2 + \quad + s_n)/n$$

and g is the geometric mean, namely,

$$g = (s_1 s_2 \quad s_n)^{1/n}$$

and this is only true when

$$s_1 = s_i \qquad i = 1, 2 \qquad n$$

For a simple proof of this theorem see Courant and Robbins,[3] p 363

On applying the theorem to the present problem one has an immediate consequence:

$$P_{T_1}/P_{T_0} = P_{Ti+1}/P_{Ti} \qquad i = 0, 1 \qquad (n - 1) \qquad (6)$$

and, therefore,

$$y_1 = y_i \qquad i = 0, 1 \qquad (n - 1) \qquad (7)$$

and other similar relations may be deduced Now $x_{i+1} = x_i f_i g_i$, and, by virtue of Eq (7), one may write

$$x_{i+1} = x_0(f_0 g_0)^{i+1} \qquad i = 0, 1 \qquad (n - 1)$$

and in particular

$$x_n = x_0(f_0 g_0)^n \qquad (8)$$

Now, because the restriction that the final shock is to be normal has been dropped, it follows from Eq (8) that y_0 will depend on M_n (or x) as well as on (M_0, n) Evidently, the closer x_n approaches x_0, the larger will be the magnitude of P_{T_n}/P_{T_0} for given (M_0,n) In fact, in the limit as $x_n \to x_0$, $(P_{T_n}/P_{T_0}) \to 1$, and again one has a trivial result However, for application to an intake, it is necessary that the flow downstream of the final shock be subsonic The limiting condition here, therefore, will be where the flow downstream is sonic, namely, $x_n = (\gamma + 1)/2$ This condition, when substituted into Eq (8), will yield for given (M_0,n) the maximum possible value of P_{T_n}/P_{T_0} for an intake shock system Hence,

$$(\gamma + 1)/2 = x_0(f_0 g_0)^n$$

Substituting for y_0, one may obtain, after a short calculation,

$$y_0 = \frac{1 + \left(\frac{(\gamma+1)/2}{1 + [(\gamma-1)/2]M_0^2}\right)^{1/n} + \left\{\left[1 + \left(\frac{(\gamma+1)/2}{1 + [(\gamma-1)/2]M_0^2}\right)^{1/n}\right]^2 - \frac{16\gamma}{(\gamma+1)^2}\left(\frac{(\gamma+1)/2}{1 + [(\gamma-1)/2]M_0^2}\right)\right\}^{1/2}}{\frac{8}{(\gamma+1)^2}\left(\frac{(\gamma+1)/2}{1 + [(\gamma-1)/2]M_0^2}\right)^{1/n}} \qquad (9)$$

This equation is quite easy to compute, and some numerical results are shown in Table 2, and in Fig 1 It will be noted that P_{T_n}/P_{T_0} is significantly higher than for the corresponding Oswatitsch shock systems

4 Concluding Remarks

In the present analysis, the perfect gas shock wave relations were retained and so also were the constraining Eqs (1) This has the advantage of permitting a direct comparison between the Oswatitsch method and the present method Equation (9) is only valid in these circumstances, and, moreover, it only applies to the limiting condition of sonic flow at the inlet In practice it is thought desirable that there should be subsonic flow at the inlet and that the final shock in the system should be nearly normal to the entry flow The margin between the total pressure recovery of the Oswatitsch series and the new series is sufficiently wide to permit the design of intakes with a subsonic inlet Mach number and yet still retain significantly higher pressure recovery Furthermore, the shock wave angle of the final wave of the new series, although not 90°, is still fairly large For example, in a high performance intake, the final wave appears in the approximate Mach number range of M_{n-1} 1 1 to 1 3, and the corresponding wave angles for a shock with sonic flow downstream are ω 73° − 65°; for no value of M_{n-1}, is $\omega_s < 61$ 3°

The derivation of Eq (6) is independent of both the perfect gas equation and the constraining equations This means, for example, that Eq (6) can be applied to the design of axisymmetric intakes The Oswatitsch theory is not valid in this case because the Mach number in the conical fields between the shocks is variable, that is, Eqs (1) are not valid Equation (6) also can be applied to the design of hypersonic intakes where the perfect gas equation is not valid and where also there may be a requirement for supersonic flow downstream of the shock system so that supersonic combustion can be used In the present context, the least restrictive problem that the method can handle appears to be the following: Given an adiabatic stream of gas that passes consecutively through a number of discrete dissipative processes, the condition for maximum total pressure recovery (or minimum entropy change) across the entire system of processes will occur when the total pressure recovery of any one individual process is the same as that of any other individual process

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